ENTROPY – KUZNETS LAW – OPTIMAL SOCIAL AND CULTURAL INEQUALITY: PARABOLIC AND HYPERBOLIC REGULARITIES

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Among problems of contemporary social sciences, two actual tasks should be singled out:

– to find the value of social (and cultural) inequality which is optimal for each given social (or cultural) system at each given stage of its evolution, i.e. the degree of inequality which is advantageous for the system's functioning and/or development;

– to find the value of the inequality of different regions (or countries) which is optimal for the entire large-scale system (e.g., the global system).

When considering both tasks in the framework of the system approach, these tasks occur identical, being reduced to the mathematical problem of optimal *distribution of the resource* among the elements of the system. The first constructive contribution to solving this problem (in application to economic systems), was made by Simon KUZNETS. That is why in 1971 he was given the Prize of the Bank of Sweden in memory of Alfred NOBEL. Kuznets was awarded this prize for his outstanding findings in the field of economic growth and its consequences.

Since that, numerous attempts were made to apply these findings (especially those ones which concern social consequences of the economic growth) in solving various concrete problems, as well as to establish due links between Kuznets' findings and other regularities in different branches of social sciences and related fields. The most impressive are the achievements of the contemporary version of the so-called '*information approach,*' which is based on the '*principle of the information maximum*' (see, e.g., Golitsyn, 1997; Golitsyn & Petrov, 1995, 2005). The essence of this principle consists in maximization of '*mutual information*' (Fano, 1951) between the system and its environment; due to such 'adaptive' behavior, the system's chances to survive are increasing. In turn, the value of the 'mutual information' is determined by *entropies* of the distributions both of the system and its environment, and their joint distribution.

In the model derived by Kuznets, one of the key concepts is the *inhomogeneity of the distribution* (e.g., the distribution of the population over different gradations of the income). At the contemporary (i.e., informational) stage of scientific development, it seems reasonable to use the informational indicators to measure this social (or cultural) inhomogeneity. Exactly such is the *entropy* (H) of the distribution of the *resource* (R) over the elements (constituents) of the system, or over its possible states. [In general, entropy is the best measure for the behavioral diversity of any system – see, e.g., Shannon & Weaver, 1949.]

There exist various versions of the link between the entropy H and the total resource R, which is available for the system considered, each version being valid for concrete set of 'boundary conditions.' (The role of the resource which is at the disposal of the system, can be played by various substances: in physical systems – by the amount of a certain matter, or energy; in economic systems – by the sum of money; in psychological systems – by the number of operations; in cultural systems – by the number of gifted persons, etc.) The most widespread situation takes place when the elements of the system are capable of *consuming the resource* in a definite range which possesses *two limits*: upper and lower. (For example, a person can spend for his needs from almost zero sum of money – to a certain restricted sum which is enough to pay for good food, good apartment, clothes, etc.) Hence, it seems reasonable to consider the social system capable of consuming the resource in a diapason between the lower limit (let it be zero) and a certain upper limit r. In such a situation, the distribution of the system's elements over different levels of the resource consumption comes (Golitsyn, 1997, p. 88-89) to the entropy

$$H = (R/r - 1) \log (1 - R/r) - R/r \log (R/r), \quad (1)$$

which can be approximated by the formula

$$H = 4 (Rr - R^2). \quad (2)$$

It is nothing else but the so-called "*parabolic dependence*." This function equals zero both at $R = 0$ and $R = r$, i.e. when the system's consumption is either minimal or maximal (in other words, when the resource available responds either to the lower limit or the upper one, i.e., when the resource results in two extreme 'levels of activation' of the system). The function possesses the only maximum at a certain 'middle' value $R = r/2$ (i.e., when the middle value of the resource available, the middle 'level of activation'). So, proceeding from the entropial considerations, we came to the 'inverted-U' dependence of ***H*** on ***R***, this dependence being quite close to those dependences which were observed by Kuznets in economic systems ('Kuznets curves').

This "parabolic law" is valid for systems of arbitrary nature: social, physical, biological, mental, cultural, etc. (of course, if the above boundary conditions are satisfied). As well, the inclination of every system to maximize its 'mutual information' with the environment, results in the tendency to approach to the *maximal value* of the entropy ***H***. The consequences of this tendency occur to be rather important.

We shall consider some of them, together with the consequences of another phenomenon – the so-called *hyperbolic distribution* (Zipf's law) which is also deduced proceeding from the maximization of the mutual information (Golitsyn & Petrov, 1995). We can divide all the variety of such consequences into groups, following two grounds for our classification:

– Phenomena which relate to a certain social system, or economical system, or cultural one (*A*) – and Phenomena of international or intercultural relations (*B*); apropos, exactly these two classes were mentioned in the beginning of this text, responding to the most actual tasks of contemporary social sciences;

– Phenomena dealing mainly with the evolutionary behavior of the system (*a*) – and Phenomena dealing with its 'static' structural regularities (*b*).

So we come to *four classes* of phenomena, each class containing various particular cases, sometimes belonging to very 'sudden' fields. We shall dwell only upon some of them.

Aa. When considering the *evolutionary trajectory* of a definite *social system*, we find inclination to quite definite shifts of optimal distribution of the resource among the system's elements, i.e., evolutionary changes in social inequality. According to formula (2), because of growing the resource available ***R*** with time, the evolution of social inequality should have an *'inverted-U' form* (i.e., parabolic behavior, though maybe slightly distorted). The inequality undergoes evolutionary change from very low degree (zero or almost zero ***H***, all members of the society are 'equal in poverty') – via great social differentiation (large values of ***H***) – again to quite low differentiation (small ***H***, most members are 'equal in wealth').

Meanwhile, in the end of the 19th century, Karl Marx wrote about "absolute and relative impoverishing of the working class," this class at that time constituting the majority of the population. His mistake is explainable: he could observe only the first, 'ascending branch' of the dependence (2), i.e., its first half responding to growing ***H***. Nowadays, most developed countries respond to the second, 'descending branch' of the evolutionary trajectory, i.e., the decreasing social inequality. Hence, the epoch of 'class battles' caused by the resource distribution within the society, is going to its end. [We should emphasize: the above considerations presuppose the existence of the upper limit of the resource consumption, meaning mainly 'material' conditions of human life, e.g., food, clothes, apartment, etc.; as for other, 'purely social' and/or mental conditions, there might be no such limits].

Ab. In each *given moment*, the *so-called Zipf's law, or hyperbolic distribution (belonging to the class of 'steady non-Gaussian distributions' – see, e.g., Petrov & Yablonsky, 1980 should describe the form of the distribution of the population over the resource consumed*. This regularity is well known. Sometimes it (or its various versions) goes under the names of Pareto, Lotka, and so on. Meanwhile (as it was mentioned above), this regularity was also deduced from the 'principle of the information maximum' (Golitsyn, 1997; Golitsyn & Petrov, 1995). In application to social, economic, and cultural systems, this regularity looks as follows. In the system considered the probability p (r***)*** that a person consumes the resource ***r*** (or more), is:

$$p(r) = C\, e^{-\beta}, \tag{3}$$

C being constant, and ***β*** – coefficient characterizing the degree of the resource deficiency: $0 \leq \beta \leq 1$. [The value of this coefficient depends on the nature of the 'compromise' between different requirements, which usually results in hyperbolic distribution, e.g., in linguistics – the compromise between the speaker's 'laziness' (his/her wish to use minimal diversity of different words in the speech communication), and the listener's 'greed': he/she wishes to receive maximal diversity. That is why, as a rule, in literary texts we deal with the value $\beta=1$, as well as in the distribution of the population over the cities and towns of the country; $\beta=2$ for the distribution of scientists over the number of their publications, and so forth. Moreover, in some studies (Petrov, 2004) the value ***β*** found empirically, was used to calculate certain 'latent conditions' in which the statistical massif observed exists, e.g., the level of moral or social development of the social groups of the population]. Hence, the optimal state of any system (if its ***β*** is not zero, i.e., a certain deficit of the resource takes place), should be '*unipolar*': it should possess only one center (and not two or more), this center consuming maximal volume of the resource. (Otherwise, e.g., if the system is bipolar, its state is not advantageous: it would spend more resource than in the above optimal, unipolar distribution.)

Therefore, the hyperbolic distribution in question serves to specify the above evolutionary regularity (i.e., the parabolic dependence, or Kuznetz law) concerning the degree of inequality in each given moment of the evolution.

The necessity to have a certain central element in any system, is in good agreement with the model of ‘*centralization*’ which was also elaborated in the framework of the information approach (Golitsyn & Petrov, 1995, 2005). According this model, each complex developing system possesses the tendency to evolve towards such state, when it possesses a certain ‘*core element*’ controlling the behavior of most other elements. All other elements occur connected with each other mainly through this central element. The phenomenon of centralization was observed in the development of various systems, from technical ones to economical, social, and ‘genuine mental’ (cultural) systems: it is simply *advantageous* (first of all, proceeding from the tendency to economize the resource, when we deal with its deficiency).

Really, as wrote G.A.Golitsyn (2000, p.41-42): “The first telephone network directly linked subscribers to one another. However, as the number of the subscribers (n) grew, the number of connections (N) grew roughly proportional to the square of n: $N = n(n-1)/2$, and the system quickly became complicated. To simplify it, central telephone posts were introduced, so that each subscriber was now connected only to the telephone exchange, and the latter connected the subscriber to all other subscribers. The number of connections decreased sharply: $N = n$. The system of commodity exchange also developed in the direction of centralization. At first one commodity was exchanged for another; but as the number of commodities grew, one (usually gold) assumed its own special status and became a universal equivalent, a kind of ‘trading center.’ First, a commodity would be exchanged for gold, and gold would be exchanged for another commodity. This drastically simplified the system of exchange relations.

One can find analogous examples in other areas: the evolution of the nervous system from diffused to centralized, the evolution of political relations from feudal fragmentation to centralized states. We observe this evolution in the spiritual sphere as well: first there was Abraham, who discovered the one and only God behind the polytheism of the pagan pantheon, and this God was responsible for the entire variety of phenomena of the material and spiritual world. <…> Newton <…> perceived a single law, the Law of Universal Gravity, behind the multitude of particular facts and laws of mechanical motion, etc. <…> On the whole, the evolution of scientific theory follows this general law of systems as well: Newton’s three laws of mechanics, which explain all the facts of classic mechanics, were later reduced to one – the principle of least action. In geometric optics, the laws of diffusion, reflection, and refraction of light were reduced to two: Fermi’s quickest path. The number of Maxwell’s equations, which embraced all the facts of electrodynamics, was first twenty, and then Hertz and Heaviside reduced them to four; and the Theory of Relativity reduced them to one.”

Numerous consequences of this centralizing tendency seem to be rather important. For example, it is simply advantageous to have maximal intellectual forces in a certain ‘central institution’ for scientific research in each given field; accumulation of industry in a certain region, appropriate concentration of cultural life, etc. (Apropos, all these distributions should be subdued to the above hyperbolic law (3); otherwise, they would not be optimal.) However, here we should take into account such important aspect, as the *nature of the resource* considered. So its seems reasonable to single out a subclass of rather specific phenomena relating to this aspect:

Ab1. Sometimes the system considered, possesses *more than one kind of resources*, which are substantial for its functioning and development. If these different resources are tightly connected (correlated) with each other, there is no need to consider them separately, in order to find the central element that is the ‘core’ one for the system: evidently, it is exactly that element, where all kinds of the resources are concentrated. (For example, the concentration of financial resource and the political power usually coincide.) However, sometimes such tight connection does not exist. In such situations, the phenomenon of ‘*multi-centricity*’ (or poly-centricity) may appear: the system occurs divided into several subsystems, each of them possessing its own ‘central core.’ (For instance, the center of political power may occur not coinciding with the center of cultural activity, e.g., in Poland most administrative resource during last centuries, was concentrated in Warsaw, whereas Krakow was the center of intellectual life, mainly artistic one.)

Ba. Turning to much more *large-scale systems*, including regional or even *global* ones, we see quite the same regularities. The *evolutionary trajectory* of each such system is subdued to the above *parabolic law* (i.e., Kuznets regularity). So at the contemporary stage, it should mean that, in particular, the economic inequality of different regions should decrease; for instance, such evolutionary ‘*equalizing*’ should take place in application to the problem of so-called ‘Rich North’ and

‘Poor South’ (meaning the contraposition of West European countries, USA, and Japan – and most African and Asiatic countries, these two ‘polar’ groups of countries possessing rather different standards of life, effectiveness of labor activity, etc.). Proceeding from the systemic point of view, the equalizing would be advantageous for the entire global system, and eventually it should come. However, two *prerequisites* are hidden behind such a conclusion:

– Firstly, we should deal really with the global system as a certain *totality* possessing its goal(s), and not with simple ‘mechanical mixture’ of several countries or regions, each of them having its own local goal(s), which are more important for its functioning and/or development than global goal(s). It’s a pity, nowadays, we see strong deviations from this precondition: some countries or even regions, being inspired by nationalistic, religious, or other motives, proclaim the priority of their own goals differing from the global ones, and neglecting these latter;

– Secondly, it worth to remind that the decreasing of inequality is advantageous for the entire system, if a certain *deficiency of the resource* is still existing. But it does really inherent in the global system, meaning primarily such resources as pure air, water, some kinds of minerals, etc.; the only thing is to realize (to reflect) the importance of this global problem.

Nevertheless, hopefully the path toward understanding both preconditions will be passed successfully, and many problems of regional inequality would be overcome. Here the most important is the good will of peoples and their governments, though of their local interests.

Bb. When speaking of *hyperbolic* (Zipfean) *distributions* in large-scale systems, we do see the phenomenon of appropriate regional concentration (and such concentration seems to be advantageous for the systems of global scale). Really, now the *world economic system* (and technical one) is *unipolar*: its central element (‘core’) is the economics of the USA. Some other systems (subsystems) tightly connected with economics, e.g., military system, of course, occur also centralized, with the same American center in the role of their ‘core.’ It is simply senseless to speak of any multi-centricity in relation to these systems. (If the global system were bipolar or multi-polar, it would be disadvantageous for the global economic situation.) However, what about other kinds of the resource (not correlated with the financial one)? Maybe, they are capable of forming subsystems differing from the previous one? This aspect needs separate consideration:

Bb1. In fact, we know some of such subsystems based on *other kinds of the resource*. Typical examples of such subsystems are some kinds of *artistic activity*. Here the ‘human resource’ is the most important factor, i.e., the concentration (in the given national culture, in the given moment) of gifted persons capable of generating innovations in the given kind of art. As it was shown quantitatively, exactly such was the evolution of European painting in the 14^{th} – 20^{th} centuries: firstly the ‘locus’ of the innovative activity (maximal concentration of talented painters) was in Italy, then it moved to France, then to Holland, and afterwards, in the beginning of the 20^{th} century, it occurred ‘split’ into two ‘sublocuses’: one in France, another in Russia (see Petrov & Gribkov, 2003). Meanwhile, the above countries were centers neither of economical nor political life at that time (maybe except some moments). Therefore, the centralization in the subsystem dealing with the *artistic activity* was *autonomous* in relation to economic or political centralization. It seems to be natural: mental life is not tightly correlated (not strictly connected) with economic activity. Hence, in the global scale we again have the possibility for ‘*multi-centricity*,’ though of unipolarity inherent in each subsystem. Therefore, in spite of the leading role of a certain country in the global economic and political life, some national cultures are capable of creating centers for special subsystems (e.g., specializing in art, or literature, or religion, etc.).

In general, the *combination of hyperbolic and parabolic regularities* (going back to Kuznets’s findings) permits to come to recommendations concerning optimization both of social and cultural life, as well as international (intercultural) relations.

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